

When the plate becomes a thin sheet we may put $\sigma = Rb$, and make b infinitely small; we shall then have

$$R \frac{dP}{dz} - 2\pi\omega \frac{dP}{d\phi} = 2\pi\omega \frac{dP_0}{d\phi}.$$

If we express P and P_0 as functions of ρ , ϕ , $z + \frac{R}{2\pi\omega}\phi$,

$$P = - \int \left(\frac{dP_0}{d\phi} \right) d\phi,$$

$\left(\frac{dP_0}{d\phi} \right)$ being found on the original system of variables. There seems to be a mistake in Maxwell's integration here, but it does not affect the representation of the result by means of the spiral trail of images which he has given.

Spherical Conductor.

In the case of a sphere or spherical shell, the vector potentials and currents will have the same general forms as before, and the solution is contained in the equations

$$\begin{aligned} \psi &= \sigma r \sin \theta \frac{d\chi}{d\theta}, & \Phi &= -\frac{d\chi}{d\phi} = -\frac{1}{4\pi} \nabla^2 P, \\ \sigma \chi &= \omega (P + P_0), & \frac{\sigma}{4\pi} \nabla^2 P &= \omega \frac{d}{d\phi} (P + P_0), \end{aligned}$$

From the general formulæ expressing completely the effect of the shell on external points, we pass to one whose thickness (c) is infinitely small: putting then $Rc = \sigma$, $\mathfrak{P} = Pr^{\frac{1}{2}}$, $Q = P_0 r^{\frac{1}{2}}$, $\rho = a \log r$, we obtain finally

$$R \frac{d\mathfrak{P}}{d\rho} - 2\pi\omega \frac{d\mathfrak{P}}{d\phi} = 2\pi\omega \frac{dQ}{d\phi},$$

which may be integrated as before. It indicates a spiral trail of images differing somewhat from the former; that due to a single pole will form a curve lying on a cone, and would be, if the cone were developed, a series of portions of an equiangular spiral.

IV. "On the Physical Constants of Liquid Hydrochloric Acid."

By GERRARD ANSDALL, F.C.S., Chemical Assistant at the Royal Institution. Communicated by Professor DEWAR, F.R.S. Received January 22, 1880.

In a paper read before the Royal Society, on June 18th, 1879, I described the results of some experiments made with liquid acetylene, obtained by compressing the gas in one of M. Cailletet's ingenious

pumps for the liquefaction of gases. The tensions of the saturated vapour then obtained were so entirely different from M. Cailletet's results, that it appeared to me to be possibly due to a slight impurity in the gas, probably a trace of mono-bromethylene which always occurs in the red copper compound of acetylene, and is very difficult to completely eliminate.

I therefore prepared a quantity of the silver compound (which can be obtained entirely free from mono-bromethylene) by passing acetylene prepared in the same way as formerly, viz., by the action of alcoholic potash on di-bromethylene, into an ammoniacal solution of nitrate of silver.

On filling several tubes with acetylene generated by the action of hydrochloric acid on this compound, the tensions were found to be exactly the same as those obtained with the acetylene made from the copper compound, thus proving the correctness of the former experiments.

As the Cailletet pump is so well adapted to the examination of gases in their liquid state, and is at the same time so easily managed, it appeared interesting to determine the physical constants of some other gases, and Professor Dewar suggested an examination of the halogen acids, with regard to their physical relations to each other, more especially as we have absolutely no reliable knowledge as to these numbers.

Faraday was the first to liquefy hydrochloric acid gas in 1823, and he subsequently, in 1848, made a series of determinations of the vapour tensions of the liquid at different temperatures. He also liquefied hydrobromic and hydriodic acids, but gives no account of their properties in a liquid state. Gore ("Proc. Roy. Soc.," vol. xiv), examined very carefully the chemical properties of liquid hydrochloric acid, its solvent action, &c., but made no determinations of its physical constants.

Hydrochloric acid is the only one I have as yet had time to examine, with the following results:—

The gas was made by the action of strong sulphuric acid on dry chloride of ammonium, being afterwards freed from sulphuric acid and dried before entering the tubes. It was found of no advantage to continue the stream of gas for more than six hours, as a constant minimum of air always remained in the tube at the end of this time (about $\frac{1}{400}$ th of the total volume of the tube) which it appeared utterly impossible to remove.

The pump was used in the same way as described in my former paper, two iron reservoirs being used, one containing the air manometer, and the other the tube with the gas to be liquefied.

Apart from the mere determination of the vapour tensions, densities, &c., the ratio of the volume of saturated vapour to that of the

liquid was considered of the highest importance, as from these numbers the latent heat of transformation and other important data can be easily calculated. For these reasons the gas was examined in rather a different way to the acetylene, the volume to which it had been compressed at the point of liquefaction (or the volume of the saturated vapour) at any given temperature being first accurately determined, and then the pressure increased until the condensed liquid entirely filled the upper part of the tube. The volume of this liquid column was then measured, so that a comparison between the volume of the saturated vapour and the volume of the total condensed liquid was obtained at each temperature.

This fractional volume of the gas at the point of liquefaction was determined by observing when the air manometer ceased to show an increase of pressure on compressing the gas; the volume of the gas at this point being then measured, and the pressure being then gradually released until the air manometer showed a slight decrease of pressure,—the volume of the gas was then again measured at this point, and the mean taken as the point of liquefaction.

The results of the whole series of experiments are recorded in a condensed form in the following table :—

A.	B.	C.	D.	E.	F.
4	137·31	$\frac{1}{38\cdot89}$	7·55	18·18	29·8
9·25	118·96	$\frac{1}{45\cdot75}$	7·90	15·05	33·9
13·8	103·50	$\frac{1}{53\cdot19}$	8·35	12·39	37·75
18·1	91·77	$\frac{1}{61\cdot17}$	8·74	10·50	41·80
22·0	81·19	$\frac{1}{70\cdot06}$	9·10	8·92	45·75
26·75	69·69	$\frac{1}{82\cdot94}$	9·50	7·33	51·00
33·4	55·75	$\frac{1}{105\cdot98}$	10·12	5·50	58·85
39·4	44·85	$\frac{1}{134\cdot33}$	10·68	4·19	66·95
44·8	36·34	$\frac{1}{168\cdot67}$	11·96	3·03	75·20
48·0	31·33	$\frac{1}{197\cdot60}$	12·00	2·61	80·80
49·4	27·64	$\frac{1}{224\cdot96}$	12·92	2·13	84·75
50·56	25·70	..	14·30	1·79	85·33
51·00	23·96				

In this table

A=temperature of gas.

B=volume of the saturated vapour at point of liquefaction.

C=fractional volume of the gas at point of liquefaction in relation to the initial volume under one atmosphere of pressure.

D=volume of the condensed liquid.

E=ratio of volume of gas to that of the liquid.

F=pressure in atmosphere.

The critical point was found to be $51^{\circ}25$ C.

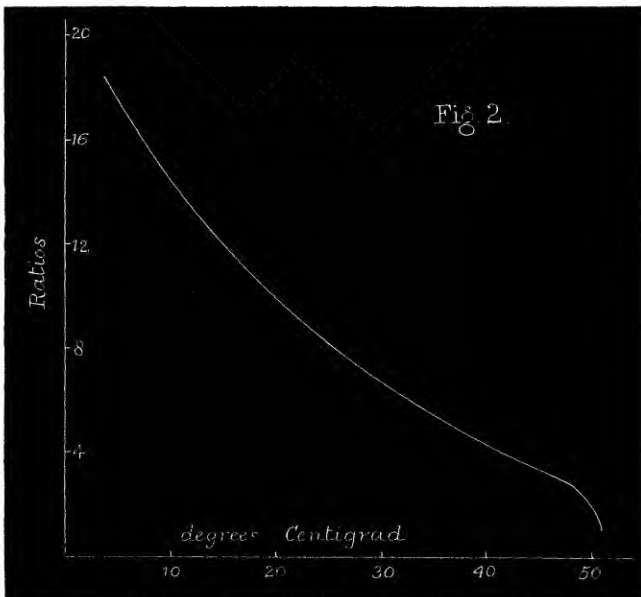
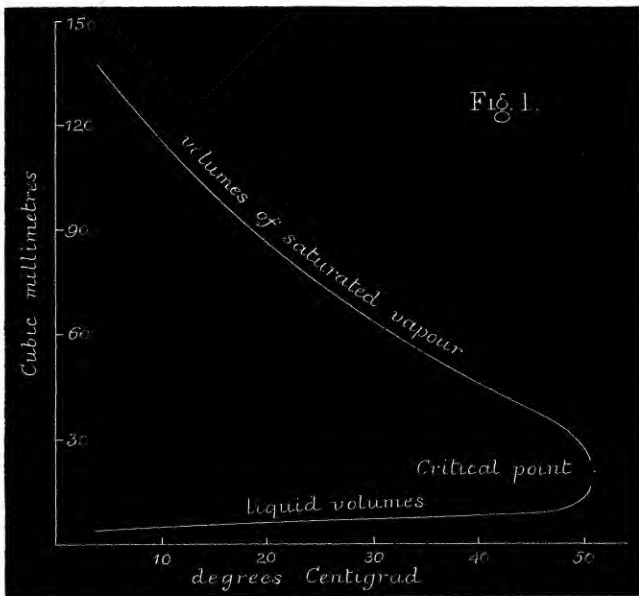
It will be seen from this table, that the volumes of the saturated vapours and liquid gradually approach each other as the temperature nears the critical point, and would undoubtedly become identical, if the experiments could be carried on up to the critical point. The following curves, the first of which represents the change of volume of the saturated vapour and liquid, in relation to the temperature; and the second the ratio between these volumes in relation to the temperature (column E in the table) show these results in a graphical form.

Curves.

The ratio between the volume of the saturated vapour and the volume of the liquid at different temperatures decreases very regularly until within about three degrees of the critical point, where a singular point in the curve occurs, and the ratio approaches unity with great rapidity. The volume of the liquid increases very regularly up to a temperature of about 48° C., and at 51° C., or within 0.25 of a degree of the critical point, the distinction between the saturated vapour and the liquid vanishes, as although liquid is plainly seen to condense on the surface of the mercury, on increasing the pressure the line of demarcation immediately disappears, and it is impossible to say whether the tube is filled with the saturated vapour or the liquid itself; therefore no results could be obtained nearer the critical point than about a fourth of a degree.

Avenarius, in a paper intitled "The Causes which determine the Critical Point" ("Mém. Acad. Sci. St. Pétersbourg," 1876-77), made a number of experiments on ether, and came to the conclusion that the volumes of the saturated vapour and of the condensed liquid at the critical point were not identical. He was under the disadvantage of working at a very high temperature, the critical point of ether being $192^{\circ}6$ C., which must have been very difficult to keep fixed within half a degree.

My own experiments, which were carried out at a very much lower temperature, and were consequently capable of greater accuracy, on account of the facility with which the temperature could be kept constant to within $\frac{1}{20}$ th of a degree, appear to confirm his results, in so far



as it is evidently impossible to measure the relative volume of fluid and gas within less than a fourth of a degree of the critical point, and at this place the volumes are certainly unequal. This, however, does not disprove their identity as the critical point.

The density of the liquid at different temperatures was determined in the same way as described in my former paper, and gave the following numbers :—

Temperature.		Density.
0° C.		·908
7·5		·873
11·67		·854
15·85		·835
22·7		·808
33·0		·748
41·6		·678
47·8		·619

This liquid has therefore not quite such a high density as liquid carbonic acid, which is 0·95 at 0° C., and is about twice as high as acetylene, which is 0·450 at the same temperature. It is interesting to note that acetylene is the lightest known fluid substance. On referring to the density of liquid hydrochloric acid given by other authors, Professors Roscoe and Schorlemmer, in their recent treatise on chemistry, describe it as a colourless liquid having a specific gravity of about 1·27. As this value is so much higher than that obtained by myself, and as I feel confident of the accuracy of my results, it seems evident that a mistake has in some way arisen, and that the value given by them refers to the density of the gaseous acid with reference to air, and not to the liquefied gas. Unfortunately Faraday does not seem to have determined its density.

Experiments made to determine the coefficients of compressibility of the fluid at different temperatures were carried on in the same way as formerly detailed. The following are the chief results :—

Mean coefficients of compression at different temperatures. Range of pressure from 52·8 to 208·19 atmospheres.

Temp. of HCl.		Coefficient.
47·0 C.	=	·00166
41·6	=	·00123
33·0	=	·00096
22·7	=	·000635
15·85	=	·00062
10·5	=	·00054
5·7	=	·000397

It is rather more compressible than liquid acetylene, whose mean coefficient at $22^{\circ}\cdot3$ C is $\cdot00058$.

The latent heat of hydrochloric acid and other values depending on thermo-dynamical considerations, will be given in a separate communication.

Presents, December 18, 1879.

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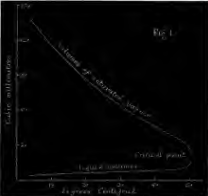


Fig. 2

